

Energy Approach to predict Uniaxial/Biaxial Load-Deformation of Woven Preforms

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ABSTRACT

The knowledge of mechanical behaviour of woven preforms under uniaxial/biaxial tensile loads is necessary to predict the changes in preform geometry during processing of composites. The aim of this study is to highlight the advantages of energy based approach to solve fabric mechanics problems without the necessity of complex 3D finite element analysis. A mechanical model to predict the tensile response of plain-woven fabric under in-plane uniaxial/biaxial loads is presented here. The model incorporates non-linear properties of constituent yarns, rather than idealised linear behaviour. All possible mechanisms of deformation including elongation, bending and compression of yarns have been considered. The predictions are compared with experimental data reported in literature and the results are discussed. The computational aspects of implementation of the model are also discussed briefly.

KEYWORDS: Mechanical behaviour, woven preforms, uniaxial/biaxial, fabric, energy

1. INTRODUCTION

Woven preforms are used extensively in composites because of their ease of handling, ability to drape and resistance to damage. In-plane tensile, shear properties, and out-of-plane bending properties are important for predicting their process behaviour such as draping and moulding into finished composites. In addition, the prediction of the deformed geometry is equally important in the micro-mechanical analysis of composites. The subject of mechanical behaviour of fabrics under uniaxial/biaxial loads has been studied by many researchers over the years. (Grosberg (1966); Freeston (1967); Kawabata (1973); Jong (1977); Hearle (1978); Haung (1979); Leaf (1980); Dastoor (1994); Potluri (2000); Boisse (2001)). A number of methods such as force equilibrium method, energy method and more recently FEM have been used to predict the tensile deformation of the fabrics. Most of the models are based on force equilibrium approach with varying assumptions regarding fabric geometry and properties of constituent yarns. The detailed literature review of various models of load-deformation behaviour of plain weave fabrics is given by Dastoor (1994). The difficulties of applying a generalised force analysis to fabric structures are numerous. It becomes necessary to make simplifying assumptions about yarn mechanical properties and points of action of forces/couples acting within the fabric structure. It is the reason why most of the models assume that the constituent yarns are linearly elastic

which is not always true because the yarn itself is a structure formed by number of filaments and the load-strain behaviour of yarn depends on many factors. Moreover different formulation of the problem for each fabric construction and for each type of deformation is necessary to use force analysis. While the Finite element Analysis offers to study the microscopic behaviour, it is computationally quite expensive and can not therefore be used in a routine manner.

With the increasing use of knowledge based CAD systems for structural design of industrial fabrics, it is appropriate to look for models which are more general in nature and can be uniformly applied for wide variety of fabric structures. The model should provide an efficient computational algorithm and handle real properties of the constituent yarns without making any simplifying assumptions a priori about yarn behaviour. The models based on energy approach offer great promise in this direction. The objective of the present paper is to show that the energy based model closely simulates the fabric mechanical behaviour and allows itself to various modifications. The advantage of energy based model is that it handles the material as well as geometric non-linearities which are the characteristic features of fabric mechanics problems. Hearle (1978) proposed an energy based approach that can be uniformly applied to the mechanical analysis of any fabric structure that is characterised by repetitive unit cell and represented by descriptive geometric model. It was shown that the method can be readily applied to

predict the load-deformation behaviour of plain woven fabrics. Some numerical results were presented for some assumed fabric parameters but no experimental verification was done.

The present work is based on energy approach used by Hearle (1978). The present model incorporates the important phenomenon of yarn compression in addition to yarn elongation and bending. The greater utility of approach is demonstrated by using modified Peirce geometry to represent yarn path. The predictions of the model are compared with the results of two other models along with the experimental data. The computational aspects of the implementation of the model are also briefly discussed. In order to get a clear understanding of the approach, the concept of energy based method and the mathematical formulation are presented in detail in the following sections. The nomenclature used in the paper is given at the end.

2. PRINCIPLES OF ENERGY BASED APPROACH

The application of energy method for solving fabric mechanics problems proposed by Hearle(1978) is based on the principle of stationary total potential energy which states that “ Of all the geometrically possible configurations which a conservative system can take up, the true one, corresponding to the equilibrium between the applied loads and the induced reactions is that for which the total potential energy is stationary.” (Richards, 1977)

The total potential ‘V’ of a fully conservative system is given by $V = U - W$

Where U is the strain energy stored in the system and W is the potential of external forces.

Mathematically, the principle of total stationary energy is expressed as

$$\frac{\partial V}{\partial q_j} = 0; \quad j = 1, 2, \dots, n$$

where ‘ q_1, q_2, \dots, q_n ’ are generalised (independent) displacements associated with the generalised forces.

In general, any mechanical system may contain one or more geometric constraints, which need to be satisfied in all configurations. If it is assumed that the geometry gives one or more relations equivalent to:

$$f_i(x_1, x_2, \dots, x_m) = 0; \quad i = 1, 2, \dots, k$$

where m = total number of arbitrary variables

k = total number of geometric constraints

It can be seen that ‘ m ’ number of variables are related by ‘ k ’ number of geometric constraints and hence the number of independent variables ‘ n ’, also called the generalised coordinates of the system, is given by

$$n = m - k$$

If the total potential ‘V’ can be explicitly expressed in terms of the generalised coordinates, the principle of stationary potential energy can be directly applied and solved for the generalised coordinates. This may not always be possible because the geometric constraints may not always be explicit functions. The problem can be overcome using the method of Lagrange. The technique of Lagrange’s method is to form a modified potential energy expression as explained below.

$$\bar{V} = V(x_1, x_2, \dots, x_m) + \sum_{i=1}^k \lambda_i f_i(x_1, x_2, \dots, x_m)$$

where $\lambda_1, \dots, \lambda_k$ are the Lagrange’s multipliers

Applying the principle of stationary potential energy, we get the relations

$$\frac{\partial \bar{V}}{\partial x_j} = 0; \quad j = 1, 2, \dots, m$$

$$\frac{\partial \bar{V}}{\partial \lambda_i} = 0 \text{ gives } f_i(x_1, x_2, \dots, x_m) = 0; \quad i = 1, 2, \dots, k$$

Thus all the variables x_1, x_2, \dots, x_m along with Lagrange multipliers $\lambda_1, \lambda_2, \dots, \lambda_k$ can be solved directly. For problems involving few variables and geometric constraints, the force-equilibrium equations can be readily derived and solved. The expressions for the geometric derivative terms can be obtained using algebraic manipulation procedure by successively differentiating the geometric constraints with respect to the chosen independent variables and solving the resulting equations. However for problems involving more variables and highly non-linear geometric constraints, it is advantageous to use suitable non linear programming techniques for direct minimisation of energy without forming explicit force-equilibrium equations. It is worth noting here that the principle of stationary potential energy does not impose any restrictions on its use for large displacement problems as long as it is possible to make reasonable assumption about the deformed configuration of the structure and be able to calculate the strain energy of constituent elements.

3. GEOMETRIC MODEL OF PLAIN WEAVE FABRIC STRUCTURE

The prerequisite to apply principle of stationary potential energy is that the fabric structure be represented by a suitable geometric model. The fabric under consideration here is a plain weave fabric formed by interlacing of two sets of yarns, called warp and weft, which are mutually perpendicular to each other. The Figure 1 shows the 3D image unit cell of plain

weave fabric and Figure 2 shows the section of a plane showing yarn path in warp direction.

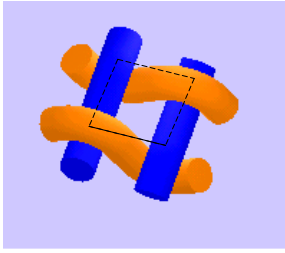


Figure 1. 3D image of unit cell of plain weave

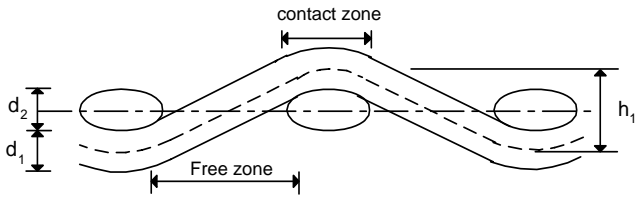


Figure 2. Section of plane showing yarn path in warp direction.

When the yarns are woven into fabric, the particular crimped form they assume for the given spacing depends on their mechanical properties i.e. bending and compression stiffness. The yarns can make distributed contact or point contact, which in turn depends on their mechanical properties. It is important that geometry of the yarn path chosen should represent the real situation as close as possible. The length of the yarn between cross over points can thus be divided into two zones i.e. contact zone and free zone. From the geometric point of view, the length of contact zone is a function of radius of contact surface and the effective thickness of warp and weft yarns at cross over and hence the cross sectional shape of two crossing yarns. Peirce (1937) made first attempt to describe yarn path by two circular arcs connected by a straight line between them as shown in Figure 3. He obtained geometric equations assuming the shape of the yarns to be circular. The only geometric constraint which needs to be satisfied in order that the two sets of yarns maintain contact with each other in all configurations is given by following equation

$$h_1 + h_2 = d_1 + d_2 \quad (1)$$

Hence it is possible to represent yarn geometry by any other regular curve as long as the geometric variables are so chosen to satisfy Equation (1). Various geometries such as twin circular arc, sinusoidal, saw tooth and elastica have been reported in the literature.

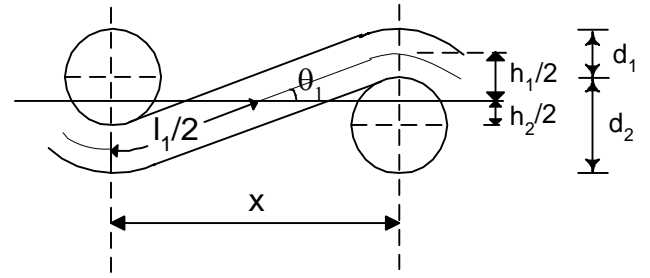


Figure 3. Peirce's geometric model of plain weave

Peirce presented his geometric model assuming the yarns to be perfectly flexible and hence the yarn path in the free zone is represented by a straight line. He has later considered the finite bending rigidity of yarns and introduced a mechanistic model describing the yarn path by an elastica. Peirce derived equations of elastica geometry assuming a point contact at the cross over of two yarns and his analysis was mainly focussed on representing the initial fabric structure. The important outcome of this model is that yarn path in the free zone also maintains some curvature due to the finite bending rigidity of yarns. Dastoor (1994) has shown that the two interlacing yarns can maintain a point contact or distributed contact in the equilibrium configuration under applied loads. Hence it is proposed to use a modified Peirce geometry by using a polynomial to represent yarn path in free zone as shown in Figure 4.

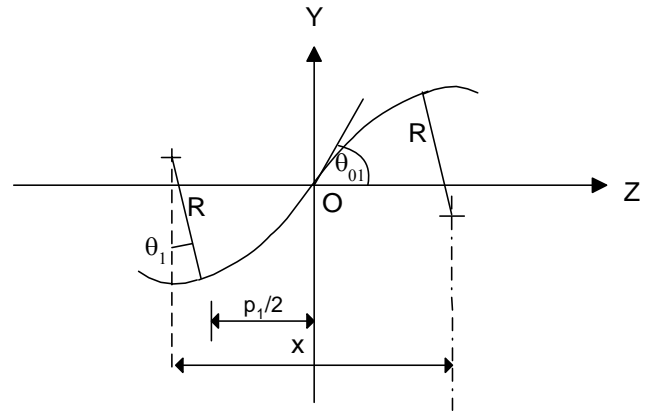


Figure 4. Modified Peirce's geometry using polynomial

The coefficients of polynomial are determined from the conditions of continuity of slope and curvature at its junction with contact zone. Hence the modified geometry satisfies the requirements of contact zone and free zone. The symmetry of plain weave requires that the point of inflexion of polynomial is at the centre of its length. The advantage of polynomial is that it is relatively easy to evaluate geometric parameters such as curvature and arc length compared to elastica. Both the Peirce geometry and modified geometry are used in the analysis and it will be shown later that the energy model based on modified geometry closely represents yarn path in real fabric. The assumption of circular shape for

the yarn is highly idealised because the yarns invariably get flattened during weaving due to inter yarn pressure. Hearle (1978) proposed a lenticular geometry using lenticular shape to represent yarn cross section. The lenticular shape which is formed by intersection of two circular arcs of equal radii represents flattened yarns and also allows the use of constant radius of curvature for the yarn path in contact zone. They have however

retained the straight line geometry to represent yarn path in free zone. Figure 5 shows the lenticular geometry. It may be noted here that the energy based approach allows to assume even other shapes such as sinusoidal for the yarn cross section as well as yarn path as long as it is possible to numerically calculate the geometric parameters such as curvature, arc length etc.,

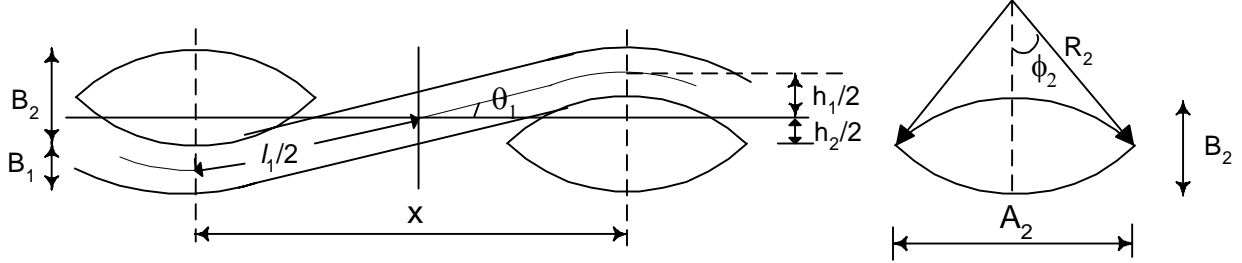


Figure 5. Lenticular geometry of unit cell of plain weave (Hearle, 1978)

4. MECHANICAL MODELLING

The complete deformation analysis of a biaxially stressed woven fabric involves the use of a large number of parameters and the consideration of many deformation mechanisms (Freeston, 1967). However the number of parameters can be reduced by treating the yarn as the basic structural unit and hence fibre properties and yarn structure do not explicitly come into picture. Fabric strains essentially result from two important phenomenon i.e crimp interchange and yarn extension. Crimp interchange results from the unbending of yarns in one direction and bending of yarns in other direction and is particularly important in low extension region. Hence the finite bending rigidity of yarns need to be considered although the tensile stiffness of yarns is predominantly high. The following important assumptions are imposed in the analysis.

1. The fabric is unset (grey) i.e the residual stresses in yarns are not eliminated by any relaxation method.
2. The deformation is homogeneous i.e. deformation of a single repeating unit, called unit cell, characterises the deformation of whole fabric.
3. The warp yarns are initially perpendicular to the weft yarns and remain so during loading.
4. The warp and weft yarns remain in contact during loading and there is no yarn slippage at yarn crossovers.
5. The yarns possess a well-defined single valued strain energy functions in elongation, bending and compression.

Considering the unit cell of plain weave fabric shown in Figure 3, it requires to specify 10 variables i.e. $x, y, l_1, l_2, \theta_1, \theta_2, h_1, h_2, d_1$ and d_2 to represent the deformed geometry. However all these variables are not

independent and they are related by the following relationships.

$$D = D_1 = D_2 = d_1 + d_2 \quad (2)$$

$$x = (l_1 - D_1 \theta_1) \cos \theta_1 + D_1 \sin \theta_1 \quad (3)$$

$$y = (l_2 - D_2 \theta_2) \cos \theta_2 + D_2 \sin \theta_2 \quad (4)$$

$$h_1 = (l_1 - D_1 \theta_1) \sin \theta_1 + D_1 (1 - \cos \theta_1) \quad (5)$$

$$h_2 = (l_2 - D_2 \theta_2) \sin \theta_2 + D_2 (1 - \cos \theta_2) \quad (6)$$

$$h_1 + h_2 = d_1 + d_2 \quad (7)$$

It can be seen that there are five equations that connect them and we need five more equations to solve all of them completely. The remaining equations can be obtained from the principle of stationary potential energy. Before proceeding further, we need to consider the mechanism of deformation. Because of difficulty of specifying a Poisson's ratio for the yarn under applied tension, the deformed diameter is found from the consideration of constant volume of the yarn for the given elongation. The deformed diameter of the yarn further gets reduced in the contact zone due to yarn compression caused by inter yarn lateral pressure and hence the radius of yarn path in contact zone depends on effective diameters of warp and weft yarns after allowing for yarn compression. The Equation (2) now takes the following form considering the constant volume of yarn and yarn compression by introducing two parameters which denote the amount of compression of warp and weft yarns respectively.

$$D = D_1 = D_2 = (d_{01}\sqrt{\frac{L_1}{l_1}} - \delta_1) + (d_{02}\sqrt{\frac{L_2}{l_2}} - \delta_2) \quad (8)$$

The problem now reduces to solving the variables i.e. $x, y, l_1, l_2, \theta_1, \theta_2, h_1, h_2, \delta_1$ and δ_2 which represent the deformed state of fabric under applied biaxial loads.

The total energy of unit cell under biaxial loads is given by

$$V = -F_x(x - x_0) - F_y(y - y_0) + U_e + U_b + U_c \quad (9)$$

The first two terms represent the potential of external loads from the undeformed configuration and next three terms represent the strain energy stored in yarns forming the unit cell due to elongation, bending and compression. U_e is computed from the load-strain (P- ϵ) curve of single yarn which can be linear or non linear. Assuming a constant strain along the length of yarn and P- ϵ curve of yarn be represented by a function $f_e(\epsilon)$, U_e is computed from the following equation.

$$U_e = \sum_{i=1}^{i=2} \left[\int_0^{\epsilon_i} f_{ei}(\epsilon) d\epsilon \right] L_i \quad \text{where } \epsilon_i = \frac{l_i - L_i}{L_i} \quad (10)$$

Youzhi Yi (1992) used average Lagrangian strain measure given by following equation to compute elongation strain energy stored in fibres presumably due to geometric non linearity of the problem.

$$\epsilon_i = \frac{1}{2} \left(\frac{l_i^2 - L_i^2}{L_i^2} \right) \quad (11)$$

It is observed that the strain energy computed using the average Lagrangian strain leads to prediction of fabric behaviour close to experimental results upto certain fabric strains. It may however be noted that use of average Lagrangian strain measure is strictly valid for small strains and leads to incorrect results if the elongation of yarn is large.

U_b is computed from the following equation using the moment-curvature (M- κ) relationship of the yarn represented by a function $f_b(\kappa)$.

$$U_b = \sum_{i=1}^{i=2} \left[\int_0^{l_i} \int_0^{\kappa_i} f_{bi}(\kappa) d\kappa \right] ds \quad (12)$$

U_c is computed from the following equation using the normal load-compression (F_c - δ) curve of the yarn represented by a function $f_c(\delta)$.

$$U_c = \sum_{i=1}^{i=2} \left[\int_0^{\delta_i} f_{ci}(\delta) d\delta \right] \quad (13)$$

It is assumed that the yarn compression which is the change in thickness of yarn under applied load can be experimentally measured as described by Kawabata (1973). Since the yarn compression is affected by the amount of tension in the yarn, Kawabata used average compression curve obtained from curves corresponding to no tension and maximum tension.

In case of relatively simple geometry such as Peirce geometry shown in Figure 3, it is possible to obtain

force-equilibrium equations using the principle of stationary potential energy. By treating five variables i.e. x, y, l_1, δ_1 and δ_2 as the independent variables, we get the following equations.

$$\left(\frac{\partial V}{\partial x} \right)_{l_1, l_2, \delta_1, \delta_2} = 0; \quad F_x + F_y \left(\frac{\partial y}{\partial x} \right) = \frac{\partial U_b}{\partial \theta_1} \cdot \frac{\partial \theta_1}{\partial x} + \frac{\partial U_b}{\partial \theta_2} \cdot \frac{\partial \theta_2}{\partial x} \quad (14)$$

$$\left(\frac{\partial V}{\partial l_1} \right)_{x, l_2, \delta_1, \delta_2} = 0; \quad F_y \left(\frac{\partial y}{\partial l_1} \right) = \frac{\partial U_e}{\partial l_1} + \frac{\partial U_b}{\partial \theta_1} \cdot \frac{\partial \theta_1}{\partial l_1} + \frac{\partial U_b}{\partial \theta_2} \cdot \frac{\partial \theta_2}{\partial l_1} + \frac{\partial U_b}{\partial D} \cdot \frac{\partial D}{\partial l_1} \quad (15)$$

$$\left(\frac{\partial V}{\partial l_2} \right)_{x, l_1, \delta_1, \delta_2} = 0; \quad F_y \left(\frac{\partial y}{\partial l_2} \right) = \frac{\partial U_e}{\partial l_2} + \frac{\partial U_b}{\partial \theta_1} \cdot \frac{\partial \theta_1}{\partial l_2} + \frac{\partial U_b}{\partial \theta_2} \cdot \frac{\partial \theta_2}{\partial l_2} + \frac{\partial U_b}{\partial D} \cdot \frac{\partial D}{\partial l_2} \quad (16)$$

$$\left(\frac{\partial V}{\partial \delta_1} \right)_{x, l_1, l_2, \delta_2} = 0; \quad F_y \frac{\partial y}{\partial \delta_1} = \frac{\partial U_c}{\partial \delta_1} + \frac{\partial U_b}{\partial \theta_1} \cdot \frac{\partial \theta_1}{\partial \delta_1} + \frac{\partial U_b}{\partial \theta_2} \cdot \frac{\partial \theta_2}{\partial \delta_1} + \frac{\partial U_b}{\partial D} \cdot \frac{\partial D}{\partial \delta_1} \quad (17)$$

$$\left(\frac{\partial V}{\partial \delta_2} \right)_{x, l_1, l_2, \delta_1} = 0; \quad F_y \frac{\partial y}{\partial \delta_2} = \frac{\partial U_c}{\partial \delta_2} + \frac{\partial U_b}{\partial \theta_1} \cdot \frac{\partial \theta_1}{\partial \delta_2} + \frac{\partial U_b}{\partial \theta_2} \cdot \frac{\partial \theta_2}{\partial \delta_2} + \frac{\partial U_b}{\partial D} \cdot \frac{\partial D}{\partial \delta_2} \quad (18)$$

It may be seen that the geometric constraint and the associated Lagrange's multiplier have not been considered in the expression for the total energy but the geometric derivative terms of Equations (14) to (18) have been derived giving regard to the geometric equations that relate them. This is done by successive partial differentiation of geometric equations given by Equation (3) to (8) with respect to the variable involved and solving the resulting simultaneous equations for the geometric derivative terms. The same methodology is used to formulate equations for the lenticular geometry shown in Figure 5 although detailed formulation is not presented here for brevity. The Equations (3) to (8) and (14) to (18) can be solved by Newton's method. Since the equations are highly non-linear, they need to be solved for the incremental load values starting from the undeformed configuration as the starting solution. Alternatively steepest descent technique can be used to predict close starting value to start Newton's method. Although it is possible to derive force-equilibrium equations for the simple geometry involving few

variables and geometric constraints, the problem gets complicated if the number of variables and geometric constraints are more. It is advantageous to use suitable non-linear programming techniques for direct minimization of total energy subject to appropriate constraints. For the modified Peirce geometry shown in Figure 4, the method of direct minimization technique has been employed. The standard library routine of NAG (Numerical Algorithm Group) for solving constrained minimization problems (E04UCFE) has been successfully used by treating total energy as the objective function and conditions of constant volume of warp yarn, constant volume of weft yarn, geometric constraint of plain weave as the non linear constraints. The equation of polynomial used to represent yarn path in free zone is given by

$$y_i(z) = a_i z + b_i z^3 + c_i z^5 \quad (19)$$

where $a_i = \theta_{0i}$

$$b_i = \left[\frac{(1 + \theta_i^2)^{\frac{3}{2}}}{3p_i R_i} + \frac{8}{3} \left(\frac{\theta_i - \theta_{0i}}{p_i^2} \right) \right]$$

$$c_i = \left[\frac{4(1 + \theta_i^2)^{\frac{3}{2}}}{5 p_i^3 R_i} + \frac{16(\theta_i - \theta_{0i})}{5 p_i^4} \right]$$

The equations of geometry of unit cell are as follows.

$$\left. \begin{aligned} R_1 &= R_2 = \frac{d_1 + d_2}{2} \\ x &= p_1 + 2R_1 \sin \theta_1 \\ y &= p_2 + 2R_2 \sin \theta_2 \\ h_1 &= 2y_1(p_1/2) + 2R_1(1 - \cos \theta_1) \\ h_2 &= 2y_2(p_2/2) + 2R_2(1 - \cos \theta_2) \\ l_1 &= 2 \int_0^{\frac{p_1}{2}} \sqrt{1 + \left(\frac{dy_1}{dz} \right)^2} dz + 2R_1 \theta_1 \\ l_2 &= 2 \int_0^{\frac{p_2}{2}} \sqrt{1 + \left(\frac{dy_2}{dz} \right)^2} dz + 2R_2 \theta_2 \end{aligned} \right\} (20)$$

The parameters $p_1, \theta_1, \theta_{01}, p_2, \theta_2, \theta_{02}, d_1$ and d_2 are treated as variables and the following non linear

constraints have been used to minimize the objective function i.e. total energy of the unit cell calculated from Equation (9)

$$\begin{aligned} d_1 - d_{01} \sqrt{\frac{L_1}{l_1}} &= 0 \quad (\text{Constant Volume of warp yarn}) \\ d_2 - d_{02} \sqrt{\frac{L_2}{l_2}} &= 0 \quad (\text{Constant Volume of weft yarn}) \\ h_1 + h_2 &= d_1 + d_2 \\ &(\text{Geometric constraint of plain weave fabric structure}) \\ &\dots\dots\dots(21) \end{aligned}$$

5. VALIDATION OF MODEL

The model is first used to represent the initial fabric structure when no biaxial loads exist. For the given spacing and diameters of yarns, the geometry of yarn path is obtained by minimizing the bending and compression energy stored in warp and weft yarns of unit cell. There are two parameters, namely 'crimp' and 'crimp height' which are used to measure geometry of yarn path. The fabric crimp in warp and weft directions is expressed separately as a ratio of difference of arc length of yarn between crossovers and spacing of yarns to the spacing of yarns which can be computed using the following equation.

$$c_1 = \frac{l_1 - x}{x} \quad \text{and} \quad c_2 = \frac{l_2 - y}{y} \quad (22)$$

Table 1 shows the comparison of results obtained using two geometries i.e Peirce geometry and modified Peirce geometry compared against experimental values reported by Ghosh (1990). It can be seen that the modified Peirce geometry using polynomial gives fabric parameters close to the experimentally measured values. The deviations from the experimental results are mainly due to neglecting the fabric set and yarn compression. However the use of Peirce geometry leads to an equilibrium configuration with zero crimp in one direction i.e. straight yarns. This shows the potential unsuitability of Peirce geometry to represent the fabric structure in some cases particularly when the bending rigidity of yarns is finite.

Table 1. Comparison of fabric crimp values predicted by energy model with experimental values

Fabric (Mono- filament yarns)	Yarn diameter 10^{-2} mm	Yarn Bending Rigidity N-mm ²	Threads No./cm		Fabric Crimp (Fractional)					
					Experimental		Energy model (Modified Peirce)		Energy model (Peirce)	
					Warp	Weft	Warp	Weft	Warp	Weft
MF1	19.75	0.092	19.3	20.1	0.0710	0.0640	0.0705	0.0781	0.0000	0.3080
MF2	19.75	0.092	19.3	16.9	0.0570	0.0530	0.0650	0.0544	0.2324	0.0000
MF5	14.50	0.037	20.5	20.7	0.0410	0.0500	0.0443	0.0448	0.0000	0.1825
MF6	14.50	0.037	24.4	23.6	0.0540	0.0620	0.0614	0.0579	0.2449	0.0000
MF7	25.75	0.319	11.8	12.2	0.0310	0.0400	0.0465	0.0484	0.0000	0.1910
MF8	25.75	0.319	15.4	15.0	0.0830	0.0450	0.0778	0.0727	0.1713	0.0243

In order to compare the prediction of energy model with regard to the load-deformation behaviour of plain weave fabric, two other models reported in the literature have been selected. Since the efficiency of model lies in predicting the behaviour of fabric under uniaxial loads rather than the biaxial loads since it involves the bending of the crossing yarns, the data of uniaxial deformation is taken for comparison.

1. Kawabata (1973) proposed a finite deformation theory to predict biaxial and uniaxial deformation of plain weave fabrics. They used idealised saw tooth geometry to represent the geometric structure. The model uses the experimentally measured tensile deformation and lateral compression properties of the yarn. Both warp and weft yarns are assumed to be perfectly flexible in biaxially loaded case where as the bending rigidity of unloaded yarns is only considered in uniaxial case.

The fabric data of fabric 1A and the yarn properties are taken from Kawabata (1973). Figure 6 shows the results obtained by energy model (circular model with yarn compression) as well as Kawabata's model compared against the experimental results. It can be seen that the energy model, which takes into account for the compressibility of yarns, predicts deformation of the fabric very close to the experimental results. It provides an efficient computational algorithm compared to Kawabata's approach where an inverse procedure is to be adopted by controlling the deformation. Moreover the consideration of bending rigidity makes the model to predict uniaxial deformation with out any additional

modifications unlike those proposed by Kawabata for uniaxial deformation case. Hence the energy model is more general in nature.

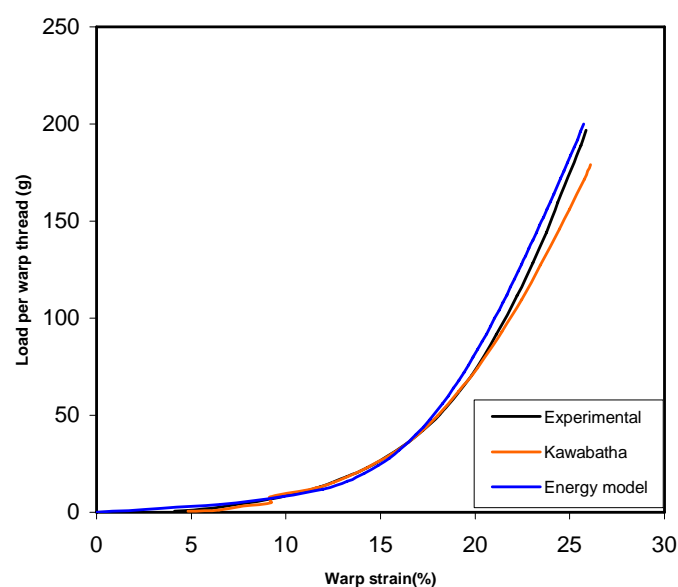


Figure 6. Comparison of uniaxial load-deformation of 1A cotton fabric of Kawabata (1973)

2. Dastoor (1994) proposed an elastica based model using force-equilibrium approach for uniaxial/biaxial load-deformation. The model assumes that the cross section of the yarn to be circular and undeformable under lateral compression while considering the finite bending rigidity and linear extensibility of the yarns. Those of MF8 fabric are shown in Figure 8. It is interesting to see that energy model predicts fabric behaviour which follows experimental curve right up to the break load due to the consideration of material non linearity of yarn unlike elastica model which assumes the yarn to be linearly elastic. The results are pretty close in case of MF2 fabric compared to MF8 fabric because the load-strain behaviour of nylon 6 yarn of MF2 fabric is fairly linear compared to that of nylon 66 yarn of MF8 fabric. However, the deviations from the experimental results cannot be easily explained due to the many factors involved but disregard of yarn compression due to the lack of experimental data may also have contributed to some extent. It may also be observed that the energy model using modified Peirce geometry gives much better results compared to energy model based on Peirce geometry. As the load is gradually increased in one direction, it results in gradual decrease of yarn crimp and gradual increase of yarn extension in that direction. But the decrease of crimp in the loading direction is associated with increase of yarn crimp in other direction and it depends very much on how the yarns bend in other direction. Hence it is very important to choose a realistic geometric model to apply energy-based approach. Since the bending rigidity of yarns of MF8 fabric is significant, the use of Peirce geometry showed significant deviations right from the low extension region. However the energy model based on Peirce geometry gives good results in case of Cotton 1A fabric shown in Figure 6 because of very low bending rigidity of cotton yarn and the comparatively low fabric strains involved. Figure 9 shows the predictions of energy model in case of MF8 fabric obtained by using average Lagrangian strain to compute elongation strain energy of yarns following Yi (1992). The prediction is more close to the experimental curve up to certain value of fabric strain but more study is needed to make valid conclusions.

The experimental data of MF2 and MF8 fabrics is taken for comparison. MF2 fabric was made from Nylon 6 monofilament 333 denier yarn while MF8 fabric was made from Nylon 66 monofilament 520 denier yarns. The results of uniaxial load-deformation behaviour of MF2 fabric are shown in Figure 7 and

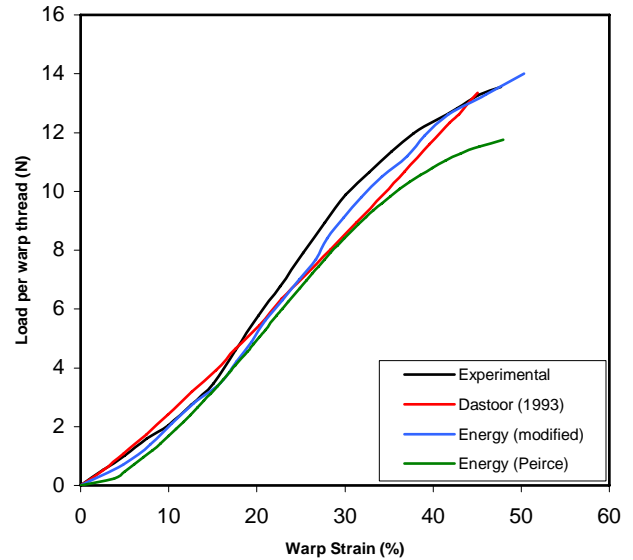


Figure 7. Comparison of uniaxial load-deformation of MF2 fabric of Dastoor (1994).

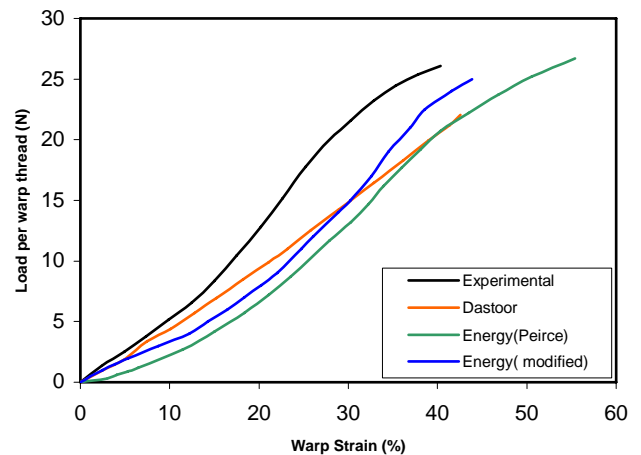


Figure 8. Comparison of uniaxial load-deformation of MF8 fabric of Dastoor (1994).

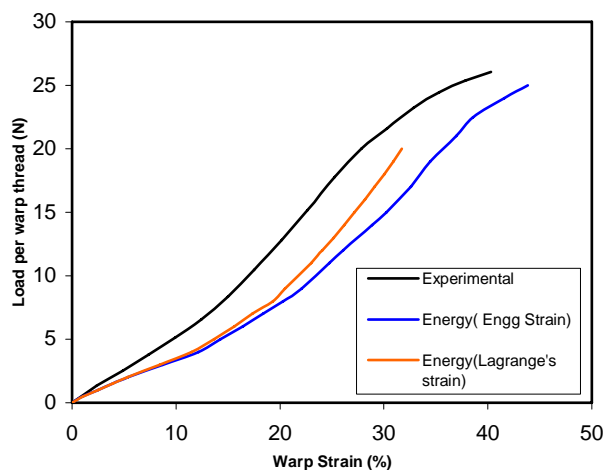


Figure 9. Comparison of uniaxial load-deformation of MF8 fabric of Dastoor (1994) using different strain measures.

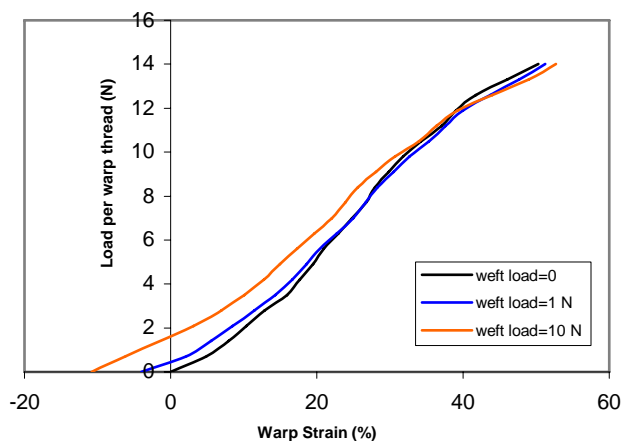


Figure 10. Effect of weft load on the warp strain of MF2 fabric of Dastoor (1994).

6. CONCLUSIONS

It can be inferred from the results that the energy based approach with a modified Peirce geometry, using a polynomial rather than a straight line yarn path, gives an improved prediction of load deformation behaviour of fabric. The prediction is consistent with the experimental results in low as well as high extension regions right up to the break load although the prediction is lower for some cases considered in the study. Since the model makes no assumption of elastic properties a priori, the measured elongation and compressibility properties can be directly used and this makes the model more realistic. It is possible to deal with complex shapes for the yarn cross section by defining the appropriate geometry to describe yarn path. Moreover the model produces an efficient computational algorithm and hence it is ideally suited for implementation in CAD systems. A software has been developed which enables to see the effect of

different parameters on the load-deformation behaviour of fabric and some results, which show the effect of weft load on the warp strain, are shown in Figure 10. It appears that the use of average Lagrangian strain to compute strain energy of yarns is consistent up to certain fabric strains. However more study is needed to make valid conclusions. The model is more general in nature and hence it can be easily extended to predict the behaviour of non plain weaves and further study is in progress to develop a generalised model, which can take into account fabric structure as a variable.

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Nomenclature

Subscript i=1 refers warp direction/yarn
 Subscript i=2 refers weft direction/yarn
 d_0 undeformed diameter of yarn
 d deformed diameter of yarn
 L undeformed length of yarn between cross overs
 l deformed length of yarn between cross overs
 x_0 undeformed spacing of yarns in warp direction
 y_0 undeformed spacing of yarns in weft direction
 x deformed spacing of yarns in warp direction
 y deformed spacing of yarns in weft direction
 θ slope of yarn path at the end of contact zone
 θ_0 slope of yarn path at the centre of its length
 R radius of yarn path in contact zone
 h crimp height of yarn between cross overs
 δ yarn compression i.e. change in thickness of yarn
 F_x load per warp thread
 F_y load per weft thread
 V total energy of unit cell
 U_e strain energy due to elongation
 U_b strain energy due to bending
 U_c strain energy due to compression
 f_e load-strain (elongation) function of yarn
 f_b moment-curvature (bending) function of yarn
 f_c normal load-compression function of yarn
 p horizontal projection of yarn path in free zone
 c yarn crimp